Geostatistical tools for analyzing spatial extreme rainfall patterns over Tunis City

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Introduction

- The identification of the structures of heavy rainfall spatial variability helps:
  - deriving rainfall patterns
  - quantifying rainfall amounts in unobserved sites
  - Areal rainfall estimations are quite important for flood and erosion risk assessment as well for economic damage evaluation

- Geostatistical tools are worth noting to address these concerns through
  - Assessment of variogram functions to identify the variability structure of a given rainfall pattern (as a random field).
  - And to perform Kriging as optimal interpolation method

Variogram assessment

- Commonly, geostatistical approach relies on the identification of the variogram function $\gamma(h)$ which represents the mean quadratic deviation in the random field
- $\gamma(h) = \frac{1}{2} E[(Z(x)-Z(x+h))^2]$

- Sample variogram $\gamma exp(h)$:
- $\gamma exp(h) = \frac{1}{2} \sum[(Z(x)-Z(x+h))^2]/N(h)$
- N(h) number of pairs for a given class of distance with average value h.

- Robust estimator (Cressie, 1993) adopts the absolute deviation
- $\gamma rob(h) = \frac{1}{2} (1/(0.457+0.494/N(h))\sum[| (Z(x)-Z(x+h)) |]$

Drawbacks of variogram function

- However the assessment of rainfall spatial dependence attached to heavy rainfall events needs specific approaches because variogram analysis assumes that the marginal distribution of the underlying random field is Gaussian
- Rivoirard (1991) proposed disjunctive kriging which corresponds to a cokriging of indicator function as a method of non linear geostatistic.
- This method was addressed as alternative to kriging of logarithmic transformation of data because of problems rising from the back transformation to original data

- Rivoirard J. (1991) Introduction au krigeage disjonctif et à la géostatistique non linéaire. Cours Centre de géologie appliquée. École des Mines de Paris.

- The gaussian type of dependence underestimates the dependence of extremes

- Cooley (2005) suggested that «the madogram $\nu(h)$ conveniently links geostatistical ideas to measures of dependence for extremes »

□ $\nu(h) = ½ \sum |(Z(x)-Z(x+h))| / N(h)$

- and that «the madogram could be used to model the spatial dependence as a function of distance, much like the variogram»
- The madogram was proposed by Matheron (1987) in linkage with the variogram of the indicator variable

- KazienKa and Pilz (2010) mentioned well-known methods to deal with non- gaussianity (kriging of logarithmic transformation of data ; disjunctive kriging; rank-order transformation)
  - Kazienka H.., J. Pilz., (2010), Copula based geostatistical modeling of continuous and discrete data including covariates. Stoch Environ Risk Assess 24: 661-673.
- Smith (1990) proposed multivariable *t*-transformation as extreme value process

- Copula function has been proposed as alternative and Bardossy and Li (2008) proposed a copula function based on Khi-Deux transformation.
  - Bàrdossy A, Li J (2008) Geostatistical interpolation using copulas. Water Resour Res 44:W07412
- **They demonstrated that Indicator kriging ; disjunctive kriging; simple kriging of the rank-transformed data are Copula based spatial interpolation models**

joint distribution (bivariate) and copula

- Copula is defined as a distribution function with marginal distributions uniforms on (0,1).
- Sklar theorem (1959) links copulas and multivariate distributions F with margines F_1 and F_2.
- $C(u_1, u_2)=\text{prob}\ (F_1(X)\leq u_1, F_2(Y) \leq u_2)$
- If the distribution is continue then the copula is unique

Aim of this work

Achieve a quantitative analysis of tail distribution of heavy rainfall eventsusing geostatistical tools

– and Generalized extreme value (GEV) bivariate distribution

- **Case study**: Analysis of Tunis daily precipitations patterns. September 17th 2003

Data

- maximum annual daily precipitation recorded at 12 stations sample sizes (16 to 116 years).
- *precipitation* records on 17/9/2003 at 75 stations max = 187 mm/day

FIG 1

Fig. 1 Réseau et totaux du 17-9-2003 (mm)

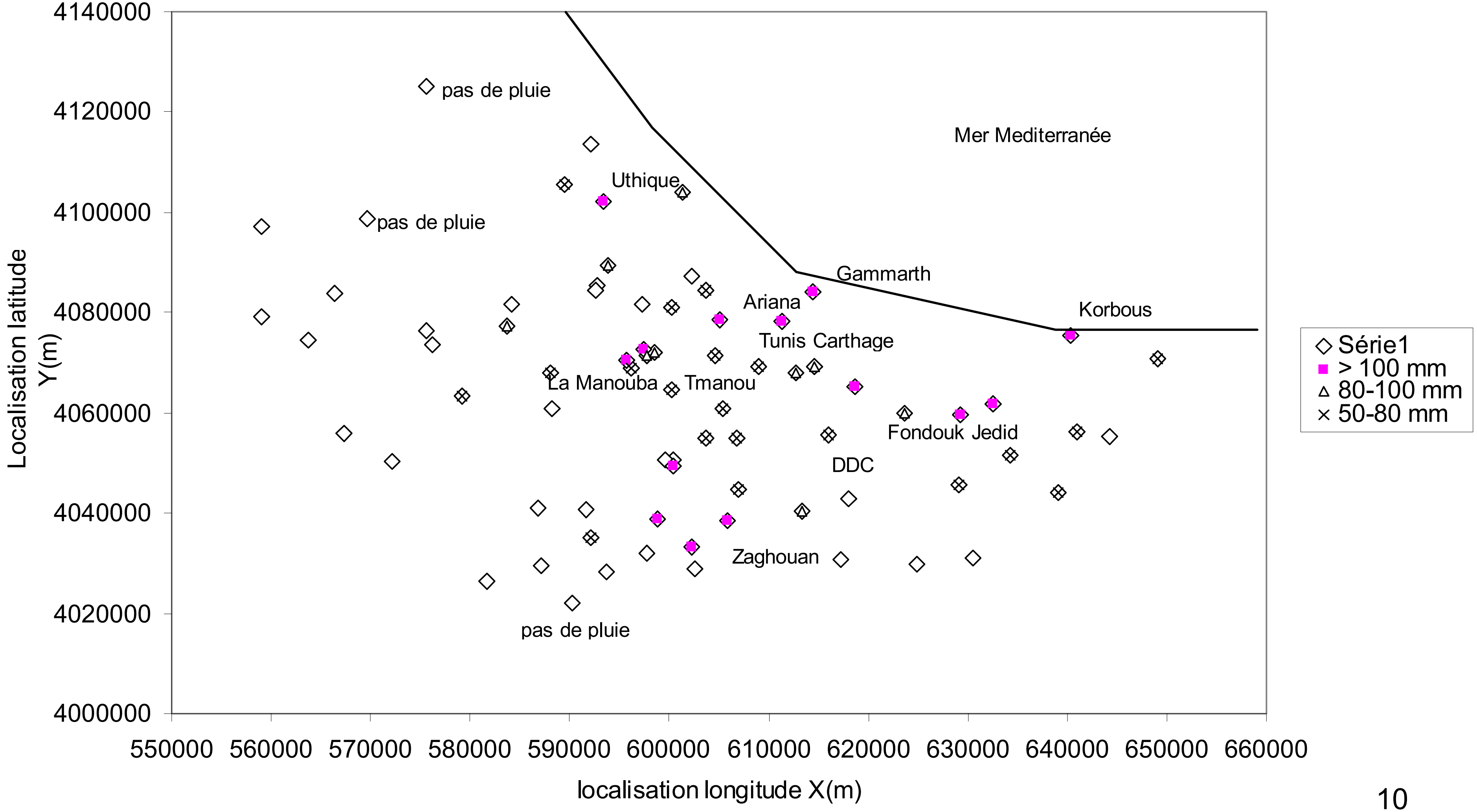
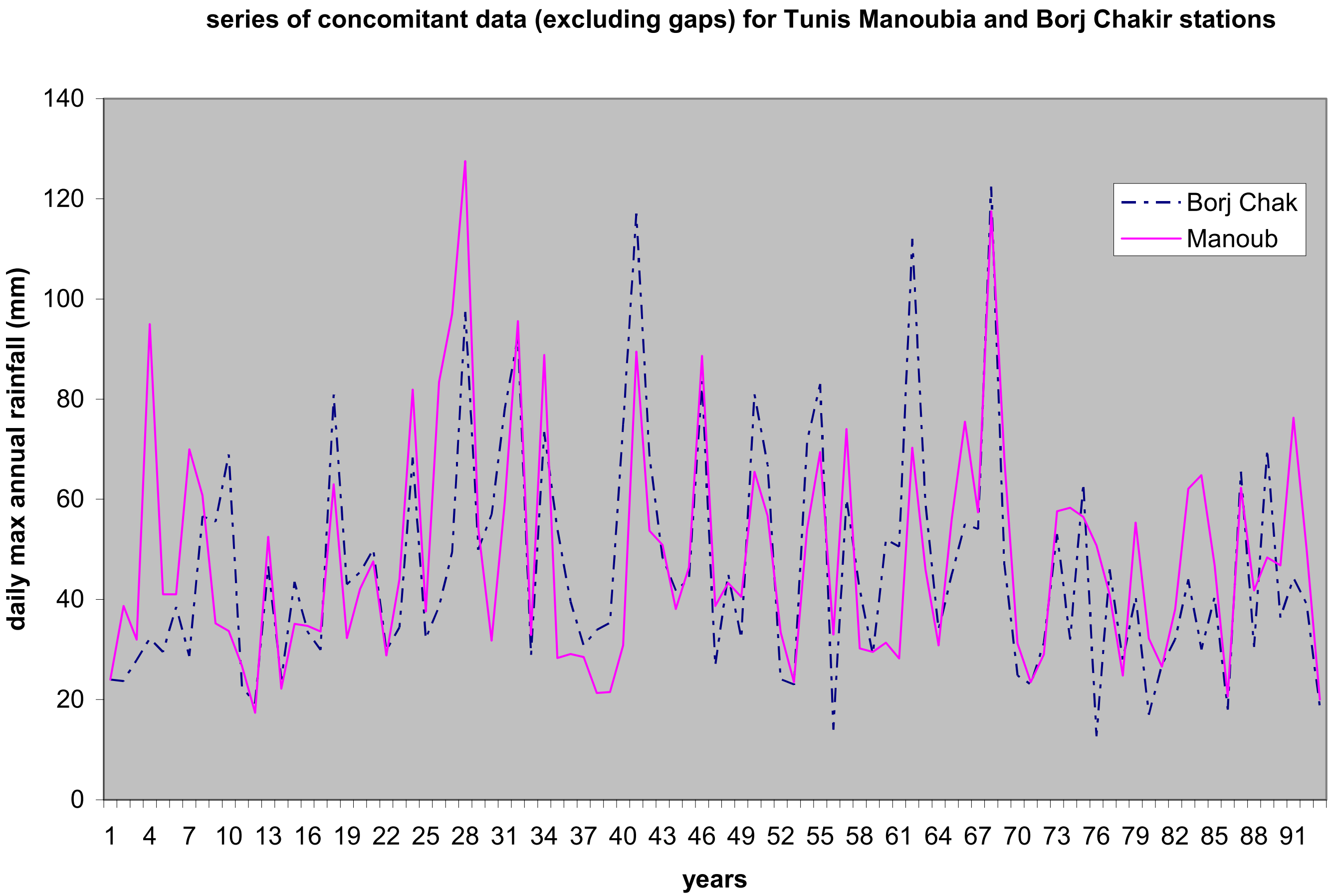


FIG 2


series of concomitant data (excluding gaps) for Tunis Manoubia and Borj Chakir stations
140
120
100
80
60
40
20
0
daily max annual rainfall (mm)
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91
years
Borj Chak
Manoub

A) Application of Extreme value theory

- $C(u,v) = \text{prob}\,(F(X) \le u,\ G(Y) \le v) = \exp\,[\log(uv)\ A\,(\log(u)/\log(uv))]$
- A is the Pickands dependence function
- (Gudendorf and Segers, 2009)

- This function has been extended in the bivariate case (Cooley;2005)
- Z(x) et Z(x+h)
- $C(u,v,h) = \text{prob}\,(F(Z(x)) \le u,\ F(Z(x+h)) \le v) = \exp\,[\log(uv)\ A\,(\log(u)/\log(uv),\ h)]$

Definition of extremal function θ

- $\theta(h)$ is defined as the tail of the joint bivariate distribution :
- **$Prob(F(Z(h)) \leq u, F(Z(x+h)) \leq u) = u^{\theta(h)}$**
- $\theta(h)$ gives a partial idea on the tail dependence structure (conversely to A);
- Extreme dependence for $\theta(h) = 1$;
- Extreme independence for $\theta(h) = 2$

Example of θ estimation

- Smith (1990) assumed local GEV model (μ,σ, ξ) for the variable X,
- he adopted a standardisation of data through the Frechet transformation

  $y = (1 - \xi(x - \mu)/\sigma))^{-1/\xi}$
- With $\Pr\{Yt \leq y\} = \exp(-1/y)$
- In this case, 1/Y has unit exponential distributions and
- 1/ $\max(y_1,y_2)$ has an exponential distribution

with mean 1/θ

- He plotted the extremal function as function of h.

Extremal dependence function and Pickands function

- In terms of the dependence function A, the extremal dependence function is (De Haan,1985)

□ **$\theta(h) = 2\,A(1/2,h)$ $(1 \leq \theta(h) \leq 2)$**

- **Etimation of A(t,h) and A(1/2,h) to derive** $\theta(h)$

Non parametric estimation of Hall and Tajvidi (2000) for A

- recommanded by Gudendorf and Segers (2009)

- $\varsigma_{HTi}(t) = \min [-\log(u)/LUM/ (1-t), -\log(v)/ LVM/ t]$; $0 \leq t \leq 1$
  - LUM average value of – log(u)
  - LVM average value of – log(v)
- Where u and v are Frechet transforms

$$1/A_{HTest}(t) = 1/k \sum_{i=1}^{k} \varsigma_{HTi}(t)$$

B) Analysis of scales of variability according to the results of the calibration of variograms and madograms

- Use of Simulated annealing (Monte Carlo simulation optimization method) with RMSE as calibration criteria.
- Changing the seed to initiate SA (replications) and multiplying the SA calibrations, it is possible to investigate the uncertainty about variogram (madogram) model.
- SA: annealing temperature decrease is according to a geometric distribution $T_n = T_0 \, (R_T)^n$
- Stopping criteria $T_n < T_{ratio}$ with $T_0 = 10^{-6}$
- R_T is computed according to the maximum number of iterations IT_{max} with no change in temperature
- $T_0 = 0.2$; $IT_{max} = 70$ so that $R_T = 0.7$.

C) Derivation of a regional GEV distribution assuming relationship between $\theta(h)$ and $\nu(h)$

- For the GEV distribution, $\theta(h)$ is related to the madogram (Cooley, 2005)
- So, $\theta(h)$ is modeled with a geostatistical spherical model.

- GEV $\xi \neq 0$:

□ $\theta(h)=[1+\xi\,\nu(h)/(\sigma\,\Gamma(1-\xi))]^{1/\xi}$

□ Γ Digamma function.

- Gumbel

□ $\theta(h) = \exp(\nu(h)/\sigma)$

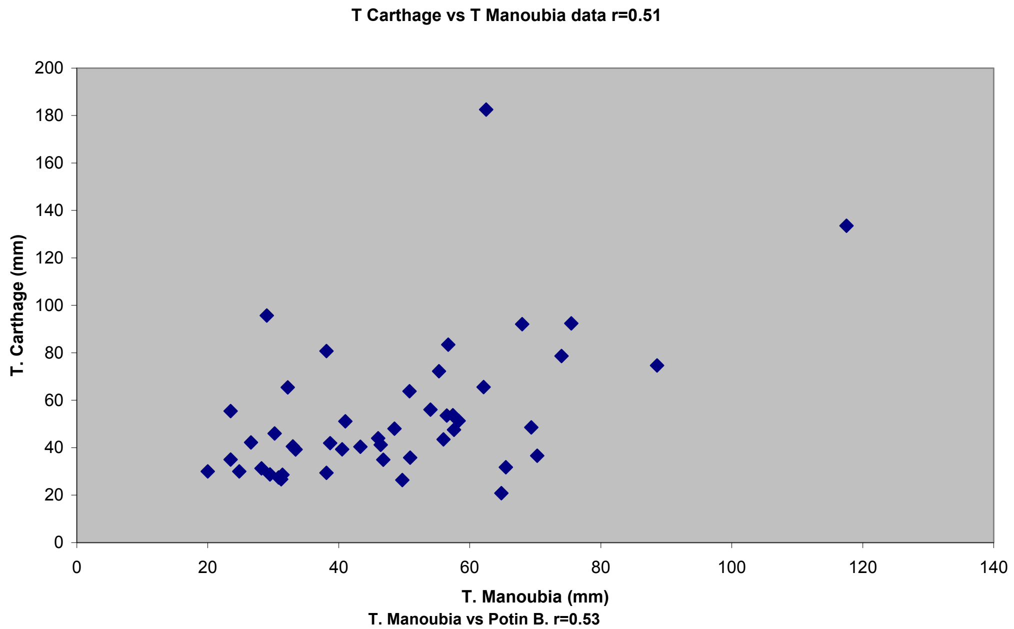
Results

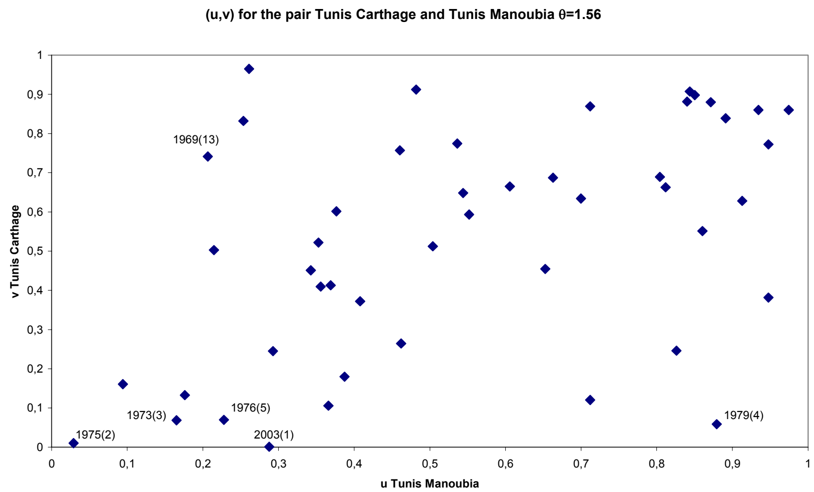
- Scatterplots (Fig. 2)
- Estimation of the dependence function (Fig. 3)
- Estimation and modeling of the extremale function (Fig. 4)
- Identification of the variogram and madogram of the rainfall field (Fig. 5a)
- Estimation of marginal GEV
- Identification of the variogram of GEV parameters separately (Fig. 5b,c)
- Comparison of scales of variability
- Mapping the conditional joint distribution for a given risk value (u=0.98) for a given location (Fig. 6)

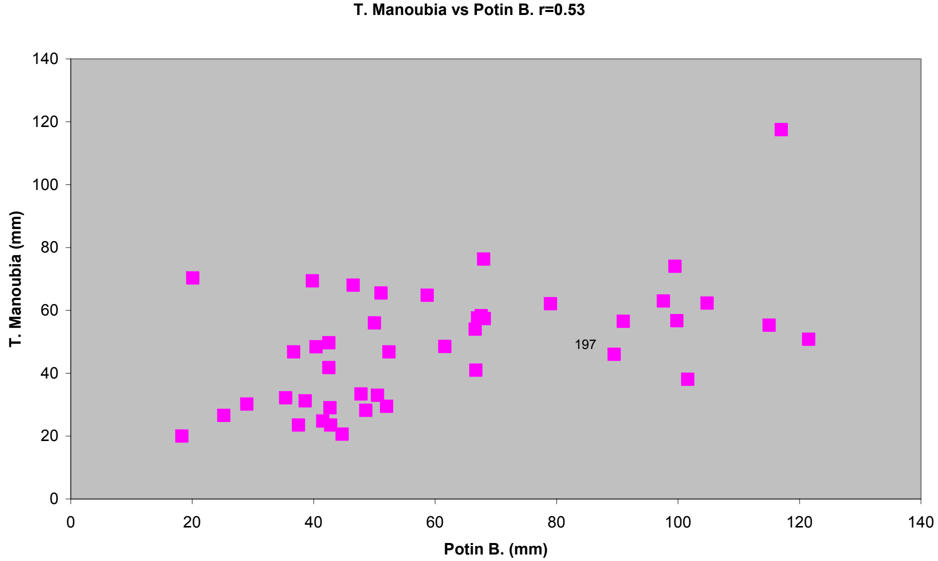
Practical aspects

- 1 km grid size (Number of nodes = 7793).
- inter distance for sampling variogram and madogram: 3.5 km.
- 18 classes of distances.

Scatterplots of data and Transformed data (FIG 2)







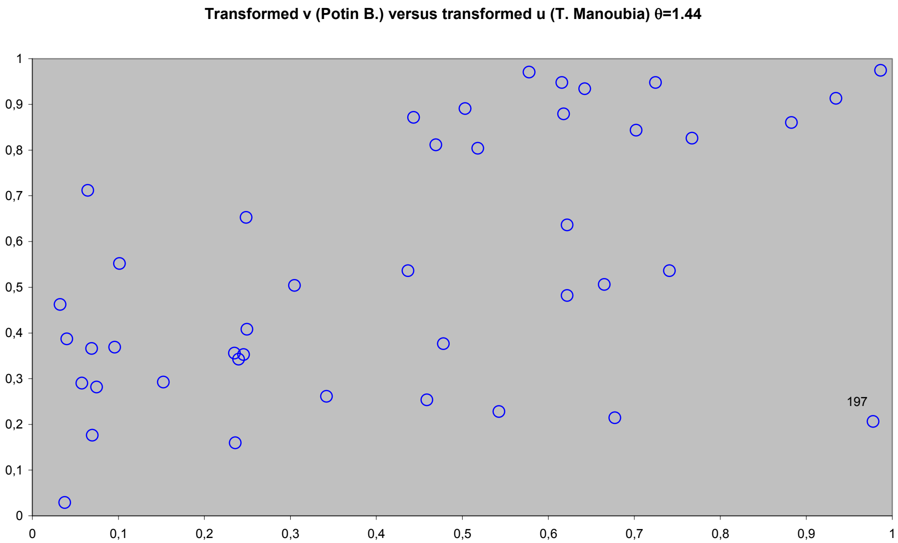
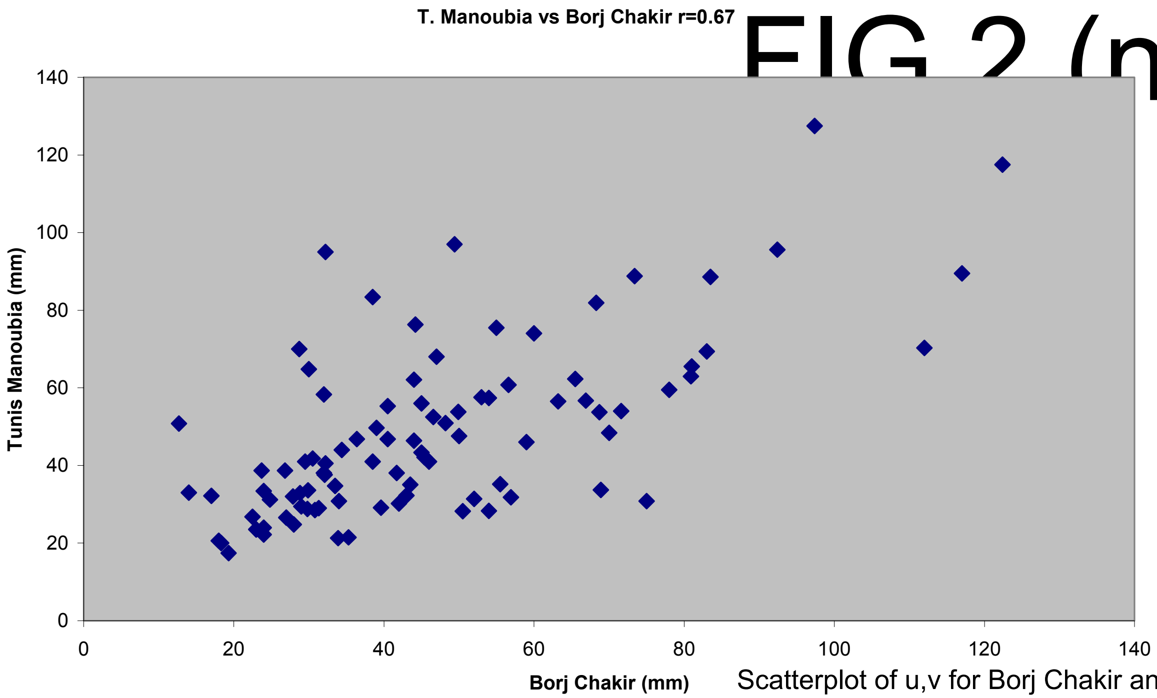


FIG 2 (next)



Scatterplot of u,v for Borj Chakir and Tunis Manoubia stations

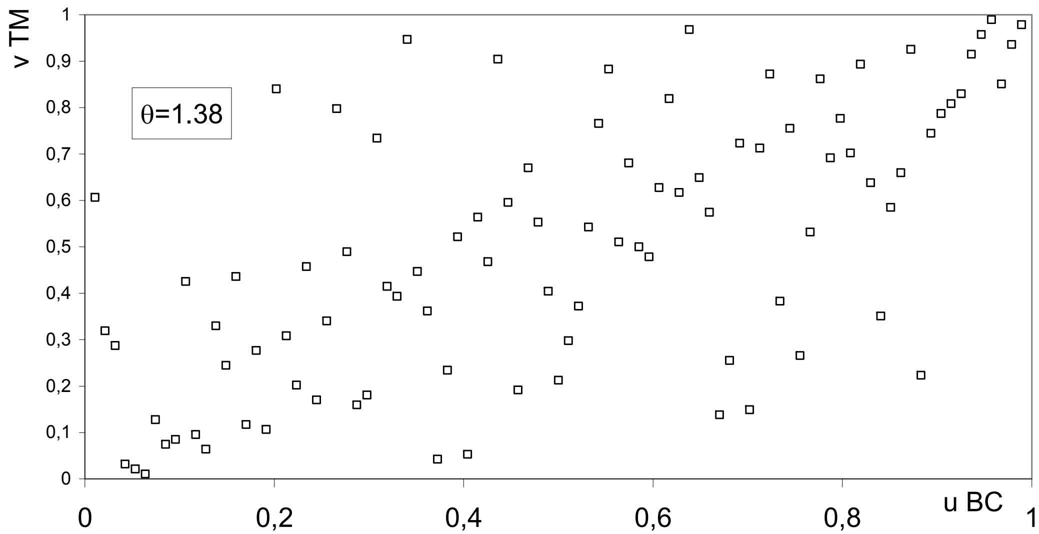


Fig. 3

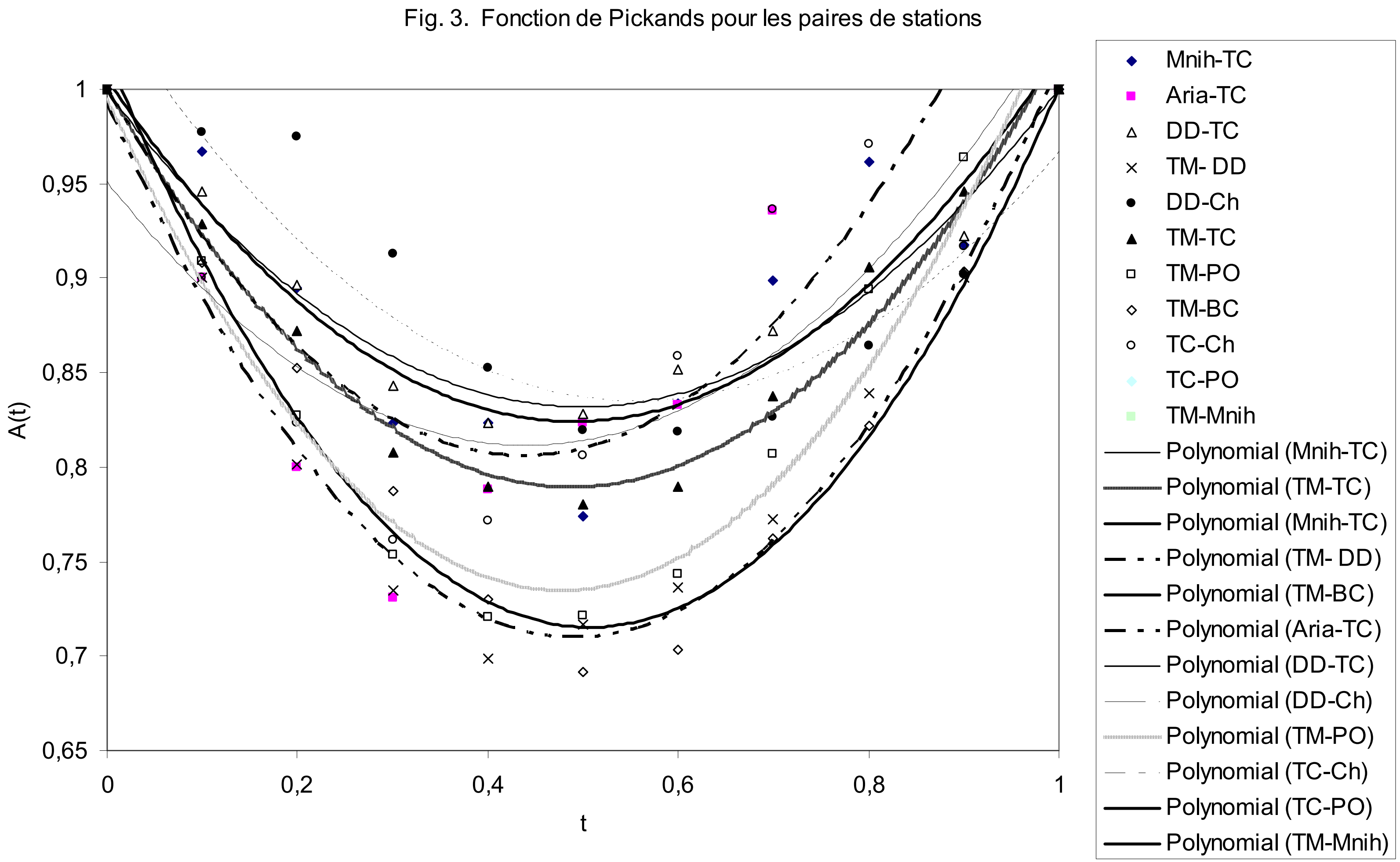
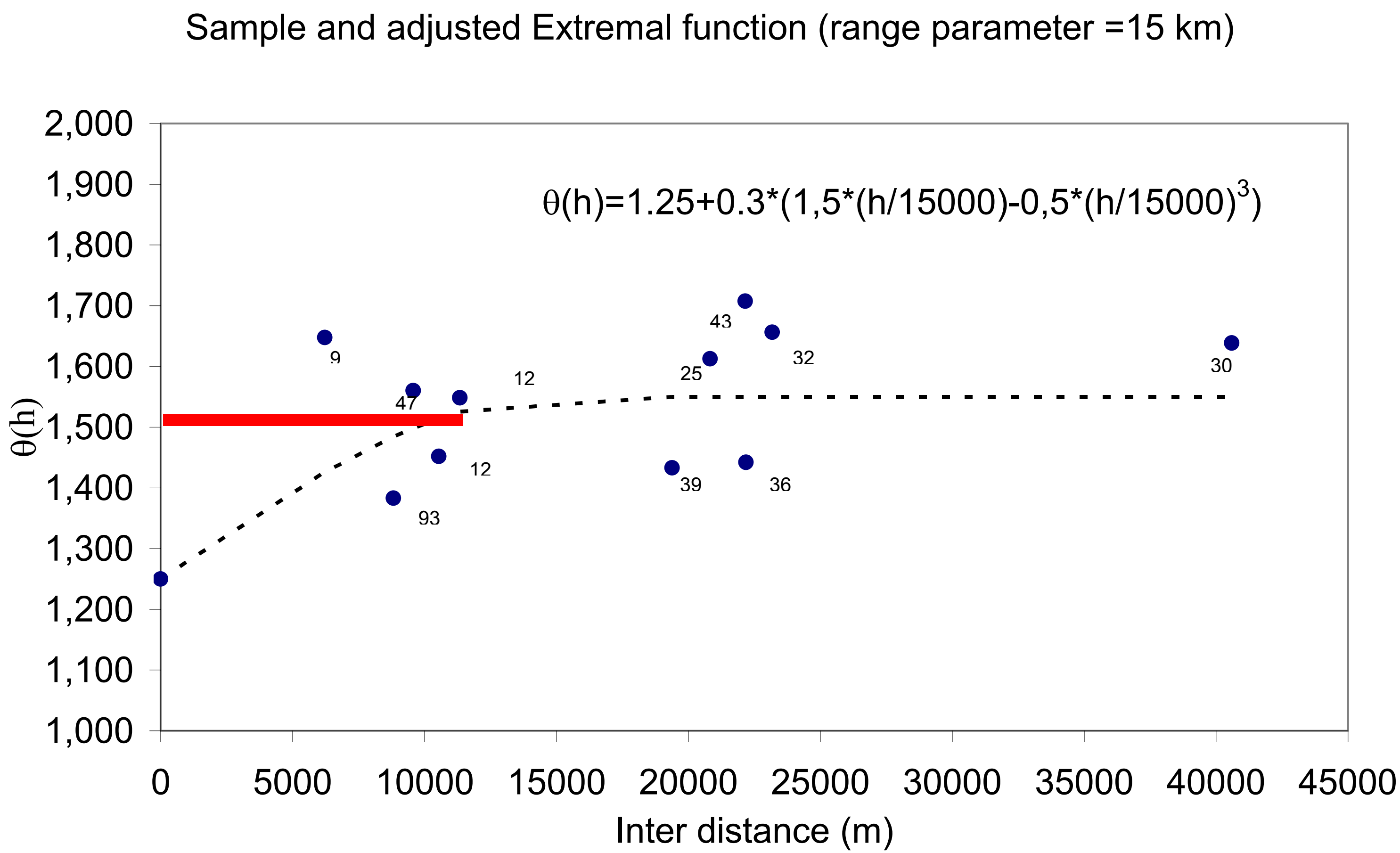


Fig. 3. Fonction de Pickands pour les paires de stations
A(t)
1
0,95
0,9
0,85
0,8
0,75
0,7
0,65
0
0,2
0,4
0,6
0,8
1
t
Mnih-TC
Aria-TC
DD-TC
TM- DD
DD-Ch
TM-TC
TM-PO
TM-BC
TC-Ch
TC-PO
TM-Mnih
Polynomial (Mnih-TC)
Polynomial (TM-TC)
Polynomial (Mnih-TC)
Polynomial (TM- DD)
Polynomial (TM-BC)
Polynomial (Aria-TC)
Polynomial (DD-TC)
Polynomial (DD-Ch)
Polynomial (TM-PO)
Polynomial (TC-Ch)
Polynomial (TC-PO)
Polynomial (TM-Mnih)

Fig. 4 (fitting of a spherical model; 10 km for extreme independency)



Partial conclusions on scales of variability according to 10 replicates of SA

- Classic and robust variograms lead to different results for the rainfall fied and the scale parameter σ field but there is insensitivity for ξ.
- Scales of variability (according to the decorrelation distance)
- Range parameter for ξ and rainfall field are of the same order (optimal values: 17 to 20 km; fluctuation of accepted solutions between 12 and 24 km).
- Conversely, for σ optimal range is 40 km and fluctuation is larger : 30 to 99 km).
- *No study of μ yet*

FIG 5

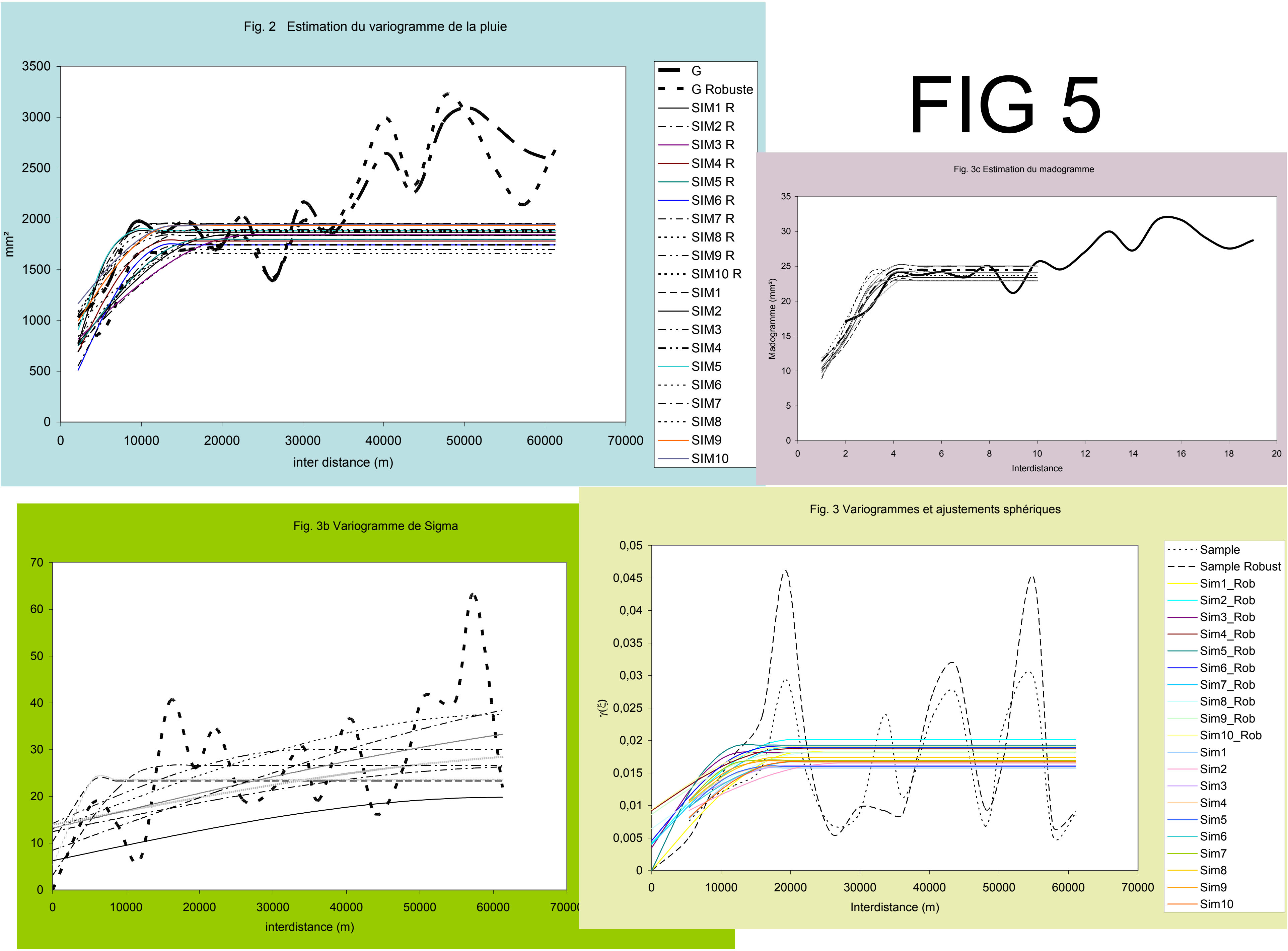


Fig. 2 Estimation du variogramme de la pluie
mm²
inter distance (m)
0
500
1000
1500
2000
2500
3000
3500
0
10000
20000
30000
40000
50000
60000
70000
G
G Robuste
SIM1 R
SIM2 R
SIM3 R
SIM4 R
SIM5 R
SIM6 R
SIM7 R
SIM8 R
SIM9 R
SIM10 R
SIM1
SIM2
SIM3
SIM4
SIM5
SIM6
SIM7
SIM8
SIM9
SIM10
Fig. 3c Estimation du madogramme
Madogramme (mm²)
Interdistance
0
5
10
15
20
25
30
35
0
2
4
6
8
10
12
14
16
18
20
Fig. 3b Variogramme de Sigma
interdistance (m)
0
10
20
30
40
50
60
70
0
10000
20000
30000
40000
50000
60000
70000
Fig. 3 Variogrammes et ajustements sphériques
γ(ξ)
Interdistance (m)
0
0,005
0,01
0,015
0,02
0,025
0,03
0,035
0,04
0,045
0,05
0
10000
20000
30000
40000
50000
60000
70000
Sample
Sample Robust
Sim1_Rob
Sim2_Rob
Sim3_Rob
Sim4_Rob
Sim5_Rob
Sim6_Rob
Sim7_Rob
Sim8_Rob
Sim9_Rob
Sim10_Rob
Sim1
Sim2
Sim3
Sim4
Sim5
Sim6
Sim7
Sim8
Sim9
Sim10

Scales of variability according to atmosphere dynamics

L_H	T	Stull	Pielke	Orlanski	Phénomènes atmosphériques
		macro	synoptique	macro-α	Circulation générale
2000km	1 mois				
		macro	synoptique	macro-β	Cyclones
200km	1 semaine				
		meso	meso	meso-α	Fronts
20km	1 jour				
		meso	micro	meso-β	Vents de montagne brises
2km	1 heure				
		meso/micro	micro	meso-γ	Circulations urbaines
200m	30 minutes				
		micro	micro	micro-α	cumulus
20m	1 minute				
		micro	micro	micro-β	panaches (cheminées), micro-turbulence

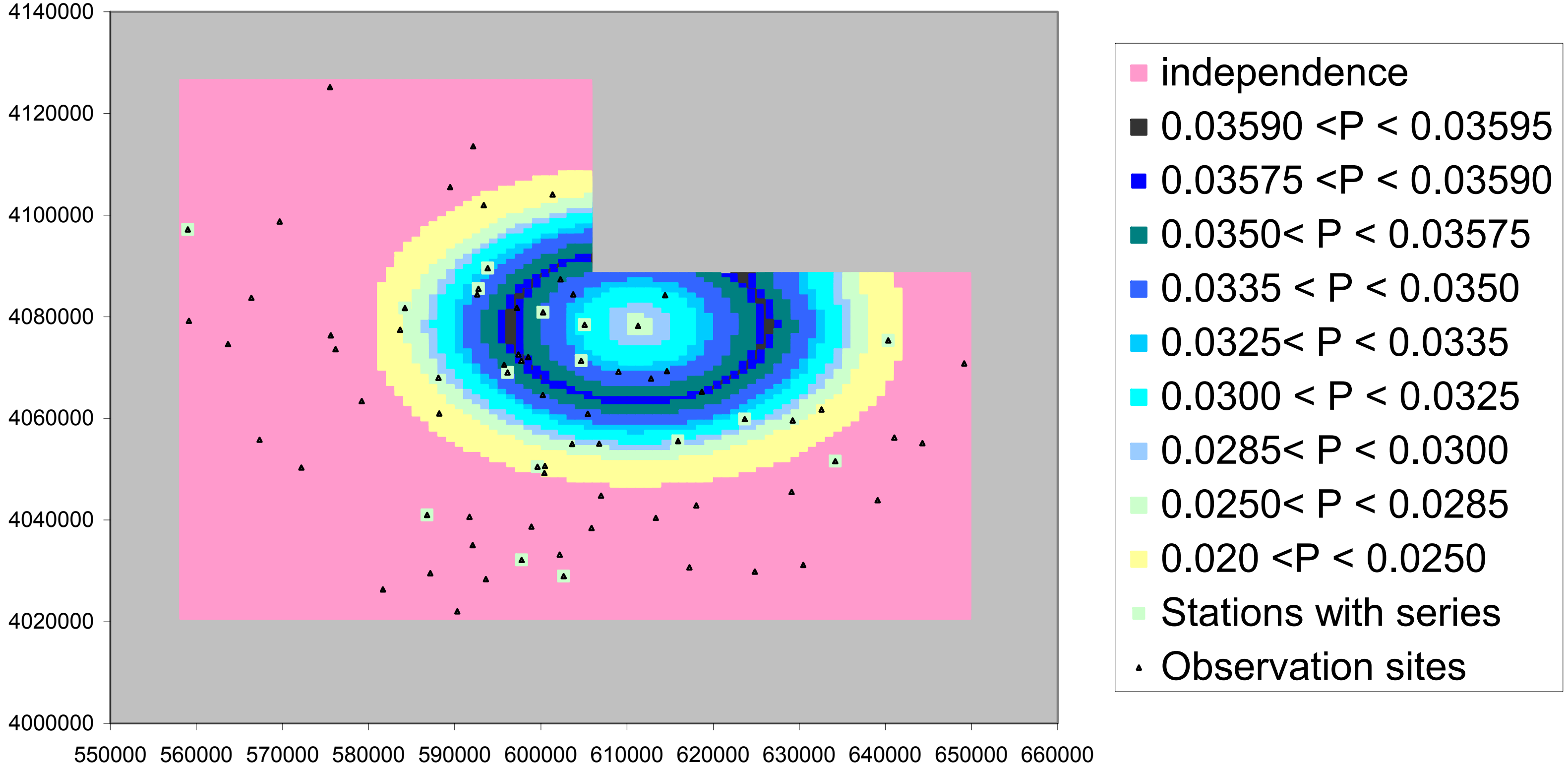
Variability of ξ and extreme 2003 rainfall field: micro scale; local effects

Variability of σ: mesoscale; dictated by General atmospheric circulation

Variability of θ (10 km) microscale effect

FIG. 6 (using the θ spherical model)

The tail of the joint bivariate GEV distribution Prob(F(Z(h))≤u, G(Z(x+h)) ≤u) = $u^{q(h)}$; Z(h):Tunis Carthage (Airport) station; u=0.98



Conclusions

- raw estimates of the extremal coefficients (not based on any model) were derived
- Estimation of standard error of θ may be achieved through Jackknife procedure (by omitting observations of a year, year by year)
- Attention is restricted to extremes based on annual maxima but the development of threshold methods should be investigated